

Mathematical analysis of sex structured population model of HIV infection in Kenya

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SAMSA Annual Conference, Palapye, Botswana

October 26, 2018

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Introduction

- HIV remains a major global health problem causing significant morbidity and mortality ([WHO, 2018](#)).
- 36.7 million people were infected with HIV/AIDS globally at the end of 2016 ([UNAIDS, 2017](#)).
 - Of these, an estimated 1.8 million were children under the age of 15 years
 - Approximately 19.8 million were women constituting about 60% of the new infections
- In Kenya, PLWHIV was estimated to be 1.5 million in the year 2017 with 28,000 deaths resulting from AIDS-related illness ([Avert, 2017](#); [UNAIDS, 2017](#))
- The HIV prevalence peaked at 10.5% in 1996 and fell to 6.0% in 2015.
- Adult prevalence (ages 15-49) is at 4.8% as at 2017 ([Avert, 2017](#)).
- Youth contributed 51% of adult HIV new infections (ages 15-24).

Introduction

- Women still continue to be disproportionately affected with HIV.
- At the end of 2016, women accounted for approximately 57% of the PLWHIV in Kenya ([Avert, 2017](#)).
- Women in Kenya face discrimination in terms of access to education, employment and healthcare.
- As a result, men often dominate sexual relationships, with women not always able to practice safer sex even when they know the risks.
- For instance, in 2014, 35% of adult women (aged 15-49) who were or had been married had experienced spousal violence and 14% had experienced sexual violence ([Avert, 2017](#)).
- Thus, there is need to study the trend of HIV infections within males and females in order to inform both on the need to limit HIV infections.

Problem statement

- There have been many studies in the literature focusing on transmission of HIV.
- These models have focused on steady states and their stability as well as computer simulations.
- Models have focused on the general population with little reference to risk groups using real-time series of HIV infection data.
- Although progress is being made to control new HIV infections, Kenya still remains one of the six high HIV burdened countries in Africa
- It is therefore desirable to study the trend of HIV infection within males and females in Kenya so as to develop a better understanding of the epidemic.

Objectives

General objective

To develop a mathematical model to integrate data on HIV within males and females in Kenya so as to develop a better understanding of the epidemic.

Specific objectives

- To develop a mathematical model that takes into account the treatment of HIV patients with ART.
- To fit the model to observed data of new infections to show the trend between males and females using the parameter values that produce the best fit to the data.
- To investigate the long term use of PrEP in the control of HIV infections.

Mathematical model

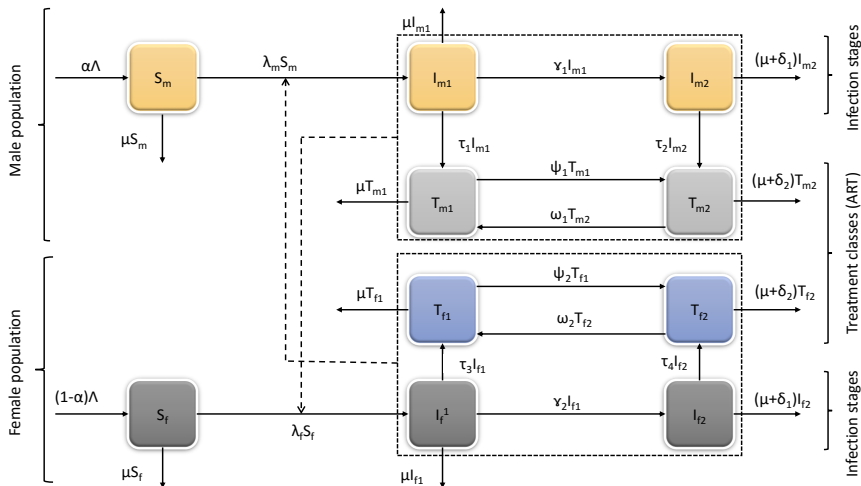


Figure: A compartmental model for the transmission dynamics of HIV, which takes into account treatment with ART.

The infection rate

$$\left. \begin{aligned} \lambda_m &= \left(\frac{\beta_{f1} c_{m1} I_{f1} + \beta_{f2} c_{m2} I_{f2} + \beta_{f3} c_{m3} T_{f1} + \beta_{f4} c_{m4} T_{f2}}{N_m} \right), \\ \lambda_f &= \left(\frac{\beta_{m1} c_{f1} I_{m1} + \beta_{m2} c_{f2} I_{m2} + \beta_{m3} c_{f3} T_{m1} + \beta_{m4} c_{f4} T_{m2}}{N_f} \right). \end{aligned} \right\} \quad (1)$$

Effect of PrEP

$$\lambda_{mq} = (1 - \phi)\lambda_m \text{ and } \lambda_{fq} = (1 - \phi)\lambda_f. \quad (2)$$

Model assumptions

- The proportion of the infected individuals on treatment is bi-directional due to attrition or adherence to ART and decline or improvement of immunological status.
- The standard HIV transmission incidence has been used to model the disease transmission. This is the form most commonly used for sexually transmitted diseases ([Hethcote, H. W. , 2000](#)).
- There is homogeneous mixing and the transmission of HIV is assumed to be mainly through heterosexual means.
- An exit due to death as a consequence of development of AIDS has been included hence AIDS class is considered redundant and thus left out.
- Furthermore, AIDS patients are usually too ill to remain sexually active and they are unable to transmit HIV through sexual activity.

Model equations

The above assumptions lead to the following ten non-linear system of ordinary differential equations.

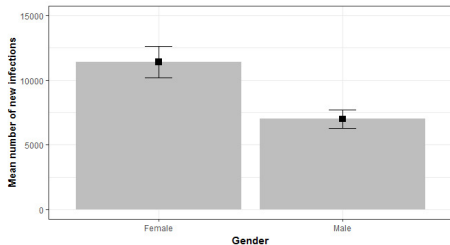
$$\left. \begin{aligned} \frac{dS_m}{dt} &= \alpha\Lambda - \lambda_{mq}S_m - \mu S_m, & \frac{dS_f}{dt} &= (1 - \alpha)\Lambda - \lambda_{fq}S_f - \mu S_f, \\ \frac{dl_{m1}}{dt} &= \lambda_{mq}S_m - (\gamma_1 + \tau_1 + \mu)l_{m1}, & \frac{dl_{f1}}{dt} &= \lambda_{fq}S_f - (\gamma_2 + \tau_3 + \mu)l_{f1}, \\ \frac{dl_{m2}}{dt} &= \gamma_1 l_{m1} - (\tau_2 + \mu + \delta_1)l_{m2}, & \frac{dl_{f2}}{dt} &= \gamma_2 l_{f1} - (\tau_4 + \mu + \delta_1)l_{f2}, \\ \frac{dT_{m1}}{dt} &= \tau_1 l_{m1} + \omega_1 T_{m2} - (\psi_1 + \mu)T_{m1}, & \frac{dT_{f1}}{dt} &= \tau_3 l_{f1} + \omega_2 T_{f2} - (\psi_2 + \mu)T_{f1}, \\ \frac{dT_{m2}}{dt} &= \tau_2 l_{m2} + \psi_1 T_{m1} - (\omega_1 + \mu + \delta_2)T_{m2}, & \frac{dT_{f2}}{dt} &= \tau_4 l_{f2} + \psi_2 T_{f1} - (\omega_2 + \mu + \delta_2)T_{f2}, \end{aligned} \right\} \quad (3)$$

subject to the following initial conditions

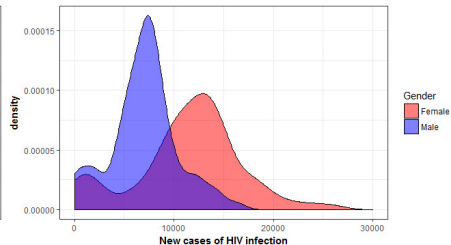
$$S_i(0) \geq 0, l_{i1}(0) \geq 0, l_{i2}(0) \geq 0, T_{i1}(0) \geq 0, T_{i2}(0) \geq 0, \text{ for } i = m, f. \quad (4)$$

$$\lambda_{mq} = (1 - \phi)\lambda_m \text{ and } \lambda_{fq} = (1 - \phi)\lambda_f. \quad (5)$$

Descriptive



(a)



(b)

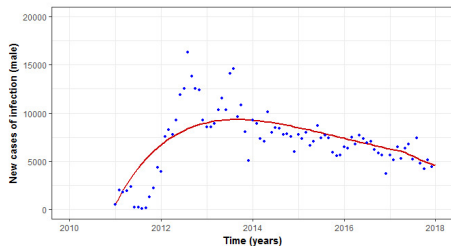
Figure: A graphical representations of the variability of the data. **2a** shows the means with error bars for two variables (females and males): $n = 84$ for each variable. The column denotes the data mean (M). The bars show confidence interval (CI). CI error bars encompass 95% of the data. **2b** shows the density distribution of the data.

Mann-Whitney U-test

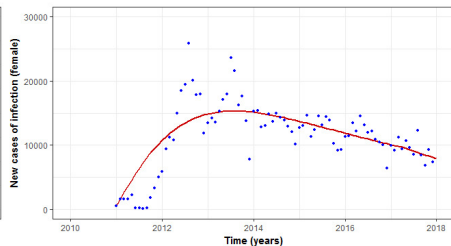
Table: Wilcoxon test.

w	p-value
5504	3.686e-10

Curve fitting



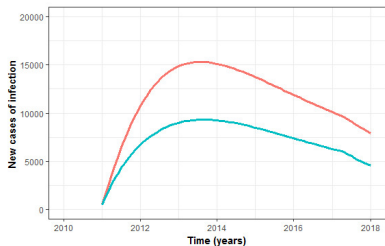
(a)



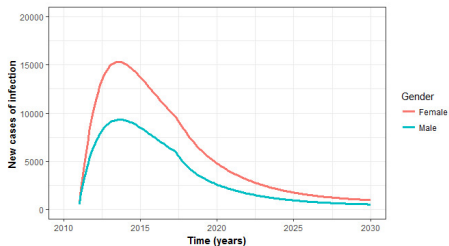
(b)

Figure: Model system (3) fitted to data for the reported new cases of HIV infection. 3a shows the model fitted to the data for the male while 3b shows the model fitted to data for the female. The blue dots indicate the actual data and the red line indicates the model fit to the data.

Curve fitting



(a)



(b)

Figure: **a** shows the comparison of the new cases of infections between male and females as fitted to the data while **b** shows the projection of infection to 2030 with a constant up-take of PrEP at 40% following its approval and its subsequent use in 2017 by the Kenyan government.

Correlation

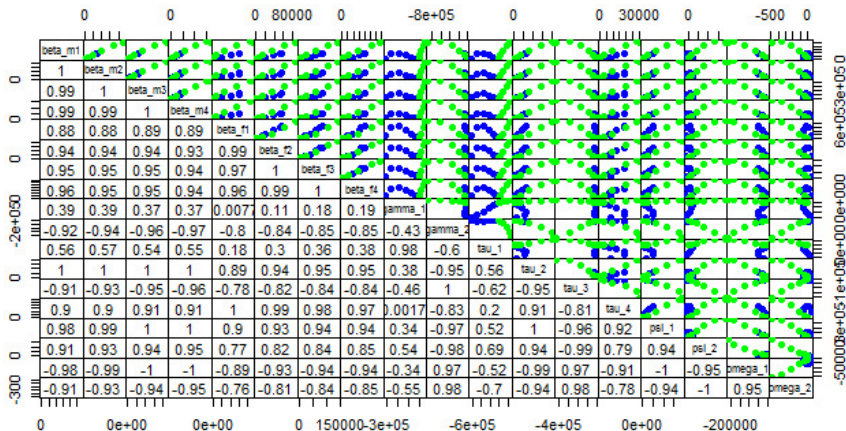


Figure: Pairs plot of the markov chain monte carlo (MCMC) samples for the model parameters.

Conclusion and future work

Conclusion

- From the trend projection to year 2030, it suffices to conclude that PrEP plays an important role in reducing the number of new cases of HIV.
- Controlling and eventual eradication of HIV in Kenya requires aggressive campaigns in favour of PrEP use.
- Prevention of HIV infection still remains the most vital way of curbing further spread

future work

- Develop a deterministic model that allows interaction within and between key risk population.
- Investigate the cost effectiveness of all the possible combinations of the two measures in order to determine the most effective strategy for eliminating HIV with minimum cost.

Key references

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Thank you